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The quantum mechanical features of a three-flavored neutrino oscillating system

Scientific research paper

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ABSTRACT

The quantum resource theory tools can examine the quantumness of neutrino oscillations across macroscopic distances. Several researchers have utilized the most popular measures of quantumness, i.e. quantum entanglement to investigate neutrino oscillations, while the quantum discord measures are rarely applied to survey this phenomenon generally due to the inherent complexity in their definition. Recently a computable measure of quantum discord is defined for bipartite systems which is generalized for multipartite systems. In this paper, we propose an alternative generalization for the bipartite measure to get rid of the maximization process in the definition of total quantum correlation measure before employing these discord measures to study the quantumness of three-flavored oscillating neutrinos both in single-particle and wave packet approaches. To support our results we compare outputs with those of a bona fide entanglement measure, i.e. concurrence fill. Although the results confirm the compatibility of the quantum discord and entanglement, the considered quantum discord measures include more quantum resources than the entanglement.

1 Introduction

Neutrinos are really interesting particles for study due to their magical properties. Neutrinos have tiny masses, participate just in weak interactions, and are neutral [1-3]. In fact, the Standard Model of particle physics predicts the existence of three types of neutrinos, i.e., ν_e, ν_μ, and ν_τ, which can be produced or absorbed alongside with their corresponding charged leptons in some weak interactions. These eigenstates are called flavor neutrinos, since any of them, corresponds to a lepton flavor, including electron, muon, or tau. Although the standard model of particle physics assumes that neutrinos are massless and left-handed particles, the evidence shows that neutrinos have nonzero mass. Neutrino oscillations would be the strongest witness for the existence of massive neutrinos. In this phenomenon, a given neutrino flavor changes into another one during its propagation. The oscillation parameters have been studied theoretically and experimentally in recent years [1-8] and revealed the quantum mechanical nature of neutrino oscillations. Investigating the quantum nature of neutrino oscillations can be done with the help of quantumness measures.

Traditionally, the quantum correlation is interpreted as the mutual relationship of two spatially separated states at the same time. This interpretation is the same concept that is discussed in Bell's inequality, while Leggett-Garg inequality (LGI) studies the correlations of a single system measured at different times based on two

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assumptions of macroscopic realism and non-invasive measurability [9, 10]. It is shown that neutrino oscillations can violate the classical limits imposed by LGI [11, 12] which is a clear indication of its quantum nature and can be taken as a measure for quantifying the quantumness of the system in the framework of quantum resource theories (QRTs) [13].

Recently, investigation of the quantumness of neutrino oscillations has been considered using various measures in the framework of QRT. A straightforward scenario to explain the time evolution of an oscillating neutrino system is the single-particle approach which is considered in several papers. Blasone et al. generalized the linear entropy for multipartite systems to study both two and three-flavored neutrino oscillations [14]. The entropy and dissension of the neutrino oscillating system are studied in Ref. [15]. Surveying three-π entanglement, concurrence fill, and concurrence triangle, Li et al. showed that concurrence fill is a genuine tripartite measure to quantify the quantumness of three-flavored electron and muon neutrino oscillations [16]. However, the limitations related to the uncertainty principle and measurement accuracy would lead one to survey neutrino oscillations in a wave packet scenario. For example, Blasone et al. benefited concurrence and logarithmic negativity to survey the quantumness of two-flavored neutrino oscillations [17]. Bittencourt et al. and Ettefaghi et al. Surveyed the effect of wave packet width on the quantumness of oscillating neutrinos using entanglement measures [18, 19].

The behavior of quantum discord measures, as a group of resources in QRT is not studied in this field, maybe because of the limited states whose discord can be computed analytically. Here we are going to examine two measures of quantum discord [20, 21] for three-flavored neutrino oscillations and compare them with concurrence fill [22].

The remainder of the paper is arranged as follows. In Section 2, we present the three-flavored neutrino oscillations model which will be considered in our research. Section 3 is devoted to a brief explanation about of concurrence fill and a computable measures of quantum discord for bipartite systems. In the rest of this section, two different generalizations of this measure for multi-partite systems are presented. In section 4, we will compare the entanglement and discord measures in the single-particle and wave packet scenarios. The paper is concluded in section 5.

2 The three-flavored neutrino oscillations model

To justify the oscillations of neutrinos, it is supposed that every weak eigenstate is a superposition of mass eigenstates ν_1, ν_2, and ν_3 [1-3].

$$|\nu_\alpha\rangle = U_{\alpha 1}|\nu_1\rangle + U_{\alpha 2}|\nu_2\rangle + U_{\alpha 3}|\nu_3\rangle. \tag{1}$$

Here we have used the Greek index for flavor eigenstates. In fact, U is a 3×3 translation matrix from mass to weak eigenstates and can be written as follows

$$U= \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{13}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}s_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\alpha_1/2} & 0 \\ 0 & 0 & e^{i\alpha_2/2} \end{pmatrix}, \tag{2}$$

where $s_{ij} = \sin(\theta_{ij})$ and $c_{ij} = \cos(\theta_{ij})$ with θ_{ij} being the mixing angles. Additionally, the phase factor δ is non-zero only if neutrino oscillation violates CP symmetry, while α_1 and α_2 are physically meaningful if neutrinos are identical to their antineutrinos.

The states $|\nu_k\rangle$ are mass eigenstates of the free Dirac Hamiltonian with energies $E_k = \sqrt{p^2 + m_k^2}$, whose time evolution can be represented as

$$|\nu_k(t)\rangle = \mathrm{e}^{-iE_k t}|\nu_k\rangle. \tag{3}$$

Suppose that $|\nu_\alpha\rangle$ is generated at the source and propagates towards a detector. As the time evolution of any mass eigenstate is different from the other ones, the phases between the terms in Eq. (1) will be changed, so one would detect another flavor in a destination. As a result, neutrino oscillation, in which an initial weak eigenstate would evolve to another one, may happen. In fact, the time evolution of a system of weak neutrinos can be shown as follows

$$\begin{pmatrix} |\nu_e(t)\rangle \\ |\nu_\mu(t)\rangle \\ |\nu_\tau(t)\rangle \end{pmatrix} = UD(t)\begin{pmatrix} |\nu_1\rangle \\ |\nu_2\rangle \\ |\nu_3\rangle \end{pmatrix} = UD(t)U^\dagger \begin{pmatrix} |\nu_e\rangle \\ |\nu_\mu\rangle \\ |\nu_\tau\rangle \end{pmatrix}, \tag{4}$$

where $D(t) = \mathrm{diag}\{\mathrm{e}^{-iE_1t}, \mathrm{e}^{-iE_2t}, \mathrm{e}^{-iE_3t}\}$. Substituting Eqs. (1) and (3) into Eq. (4), we can find the time evolution of flavor neutrino states as follows

$$|\nu_\alpha(t)\rangle = A_{\alpha e}(t)|\nu_e\rangle + A_{\alpha\mu}(t)|\nu_\mu\rangle + A_{\alpha\tau}(t)|\nu_\tau\rangle, \tag{5}$$

with $A_{\alpha\beta} = \sum_{k=1}^{3} U_{\alpha k}\mathrm{e}^{-iE_kt}\,U_{\beta k}^*$. Defining the occupation number states as follows

$$|\nu_e\rangle \equiv |1\rangle_e \otimes |0\rangle_\mu \otimes |0\rangle_\tau = |100\rangle, \tag{6}$$

$$|\nu_\mu\rangle \equiv |0\rangle_e \otimes |1\rangle_\mu \otimes |0\rangle_\tau = |010\rangle, \tag{7}$$

$$|\nu_\tau\rangle \equiv |0\rangle_e \otimes |0\rangle_\mu \otimes |1\rangle_\tau = |001\rangle, \tag{8}$$

Eq. (5) can be rewritten as

$$|\nu_\alpha(t)\rangle = A_{\alpha e}(t)|100\rangle + A_{\alpha\mu}(t)|010\rangle + A_{\alpha\tau}(t)|001\rangle. \tag{9}$$

It can be easily shown that the transition probability for $\nu_\alpha \to \nu_\beta$ is

$$P_{\nu_\alpha\to\nu_\beta} = \langle \nu_\alpha(0)|\nu_\beta(t)\rangle = \sum_{k,l=1}^{3} U_{\alpha k}^* U_{\alpha l} e^{-i(E_k-E_l)t} U_{\beta k} U_{\beta l}^*. \tag{10}$$

For the ultra-relativistic neutrinos being studied, we can use the approximation $E_k - E_l \sim \frac{m_k^2-m_l^2}{2E}$ with E being their energies. Replacing time (t) with distance (L) in Eq. (6) will lead us to

$$P_{\nu_\alpha\to\nu_\beta} = \sum_{k,l=1}^{3} U_{\alpha k}^* U_{\alpha l} \exp\left(-i\Delta m_{kl}^2 \frac{L}{2E}\right) U_{\beta k} U_{\beta l}^*. \tag{11}$$

So far, we have investigated the neutrino oscillations in a single-particle framework as a three-level system problem. Now we try to examine the effects of localization on the oscillation patterns through a simplified one-dimensional wave packet prescription. The corresponding wave packet, $\psi_k(x,t)$, for the mass eigenstate $|\nu_k(\mathrm{t})\rangle$ will be

$$\psi_k(x,t) = \langle x|\nu_k(\mathrm{t})\rangle = \frac{1}{\sqrt{2\pi}}\int \mathrm{d}p\psi_i(p)\exp[i(px - E_k(p)t)]. \tag{12}$$

For simplicity, suppose that the momentum distribution of the neutrino wave packet is a Gaussian function as follows

$$\psi_k(p) = \frac{1}{\sqrt[4]{2\pi\sigma_p^2}}\exp\left(-\frac{(p-p_k)^2}{4\sigma_p^2}\right). \tag{13}$$

Following the procedure similar to the single-particle approximation, we can define the localized density matrix corresponding to an initial $|\nu_\alpha\rangle$ as follows

$$\rho_\alpha(L) = \int dt\rho_\alpha(L,t). \tag{14}$$

It is shown that this localized density matrix can be written as follows [23]

$$\rho_\alpha(L) = \sum_{k,l=1}^{3} U_{\alpha k}^* U_{\alpha l}\,\mathcal{F}_{kl}(L)\,U_{\beta k}U_{\beta l}^*, \tag{15}$$

with

$$\mathcal{F}_{kl}(L) = \int dt\psi_k(L,t)\psi_l^*(L,t) \approx \exp\left[-i\Delta m_{kl}^2\frac{L}{2E} - \left(\frac{\sigma_p\Delta m_{kl}^2 L}{2\sqrt{2}E^2}\right)\right]. \tag{16}$$

Using the transitional matrix U we can represent the localized density matrix in terms of the calculational bases defined in Eqs. (6)-(8) as follows

$$\rho_\alpha(L) = \sum_{k,l}\sum_{\beta,\gamma} U_{\alpha k}^* U_{\alpha l}\,\mathcal{F}_{kl}(L)\,U_{\beta k}U_{\beta l}^*|\nu_\beta\rangle\langle\nu_\gamma|, \tag{17}$$

where summation on i and j is over $\{1,2,3\}$ and on β and γ is over $\{e,\mu,\tau\}$. Similar to the single-particle approach, we can easily find the corresponding density matrix of an initial flavor neutrino. In this scenario, the transition probabilities will be altered as follows

$$P_{\nu_\alpha \to \nu_\beta} = \sum_{k,l=1}^{3} U_{\alpha k}^* U_{\alpha l} \mathcal{F}_{kl} U_{\beta k} U_{\beta l}^*. \tag{18}$$

3 Quantum correlation measures

In this section, we are going to use the framework of QRT to study the three-flavor neutrino oscillation model. Finding analytical solutions for quantum discord measures cannot be done in the general case and is limited to a few specific states. For this reason, the quantumness of a general neutrino oscillating system has not been studied using quantum discord measures to the best of our knowledge. In this section, we intend to introduce two multi-partite quantum discord measures [20, 21] which can always be computed in order to take advantage of them in surveying the quantumness of the model. To compare the performance of these measures with the entanglement alternatives, we will use concurrence fill [22], which is the best measure of entanglement for studying neutrino oscillations according to the results of reference [16]. In the rest of this section, we provide a brief explanation about these three measures.

3.1 Concurrence fill

Reference [22] presented a triangle with side lengths each one corresponding to the concurrence between one qubit and the remaining two taken together as the "other" single party. The concurrence fill is defined as the square root of the area of the concurrence triangle. It is shown that the concurrence fill can be expressed as follows

$$F_{ABC} = \sqrt[4]{\frac{16}{3}\xi(\xi - C_{A|BC}^2)(\xi - C_{B|AC}^2)(\xi - C_{C|AB}^2)}, \tag{19}$$

where

$$\xi = \frac{1}{2}\left(C_{A|BC}^2 + C_{B|AC}^2 + C_{C|AB}^2\right), \tag{20}$$

3.2 Computable measure of quantum correlation

This quantum discord measure is based on the so-called A-correlation matrix of a generic bipartite state ρ on $\mathcal{H}^{d_A} \otimes \mathcal{H}^{d_B}$ which is defined as follows [20]

$$\mathcal{T}^A = \sqrt{\frac{2}{d_A^2 d_B}} \begin{pmatrix} f_1(y)\vec{x} & f_2(y)\sqrt{\frac{2}{d_A}}T \end{pmatrix}, \tag{21}$$

where $\vec{x} = (x_1 \quad x_2 \quad \cdots \quad x_{d_A^2-1})$ is the coherence vector of density matrix of subsystem A (ρ^A) whose elements are defined as

$$x_i = \frac{d_A}{2} Tr\left[\left(\hat{\lambda}_i^A \otimes \mathbb{I}^B\right)\rho\right], \tag{22}$$

with $\{\hat{\lambda}_i^A\}_{i=1}^{d_A^2-1}$ being the generators of $SU(d_A)$, $f_1(y)$ and $f_2(y)$ are nonzero functions of $y = \sqrt{\vec{y}^t\vec{y}}$ with $\vec{y} = (y_1 \quad y_2 \quad \cdots \quad y_{d_A^2-1})$ being the coherence vector of the density matrix of subsystem B (ρ^B) whose elements are defined as

$$y_j = \frac{d_A}{2} Tr\left[\left(\mathbb{I}^B \otimes \hat{\lambda}_j^B\right)\rho\right], \tag{23}$$

with $\{\hat{\lambda}_j^B\}_{j=1}^{d_B^2-1}$ being the generators of $SU(d_B)$, and T is the correlation matrix of the system with the following elements

$$T_{ij} = \frac{d_A d_B}{4} Tr\left[\left(\hat{\lambda}_i^A \otimes \hat{\lambda}_j^B\right)\rho\right]. \tag{24}$$

The measure is defined as follows

$$\mathcal{Q}_A = \min_{P_A} \|\mathcal{T}^A - P_A\mathcal{T}^A\|_2^2, \tag{25}$$

with P_A being a $(d_A - 1)$-dimensional projection operator on $(d_A^2 - 1)$-dimensional space. It is shown that [20]

$$\mathcal{Q}_A = \sum_{k=d_A}^{d_A^2-1} \tau_k^{A\downarrow}, \tag{26}$$

where $\{\tau_k^{A\downarrow}\}$ are the eigenvalues of $\mathcal{T}^A(\mathcal{T}^A)^\dagger$ in non-increasing order.

A straightforward way to generalize this measure for a generic N-partite state $\rho^{A_1A_2\cdots A_N}$ is to calculate the corresponding quantum discord of any subsystem $\rho^{A_i|\{A_1A_2\cdots A_{i-1}A_{i+1}\cdots A_N\}}$ and average over all of the results as follows

$$\bar{Q}_{A_1A_2\cdots A_{i-1}A_{i+1}\cdots A_N} = \frac{1}{N}\sum_{i=1}^{N} Q_{A_i|\{A_1A_2\cdots A_{i-1}A_{i+1}\cdots A_N\}}. \quad (27)$$

In this research, we chose the simple functions choice of $f_1 = f_2 = \frac{\sqrt{2}}{2}$ which normalizes the geometric discord-like measures. As a result, for the tripartite density matrix of the three-flavored neutrino oscillating system, we have

$$\bar{Q}_{A_i|A_jA_k} = \frac{2}{3}\left(Q_{A_1|A_2A_3} + Q_{A_2|A_1A_3} + Q_{A_3|A_1A_2}\right). \quad (28)$$

3.3 Computable measure of total quantum correlations

An alternative generalization for the quantum discord measure Q_A is provided in reference [21]. A generic tripartite state ρ on $\mathcal{H}^{d_{A_1}} \otimes \mathcal{H}^{d_{A_2}} \otimes \mathcal{H}^{d_{A_3}}$ can be represented as follows

$$\rho = \sum_{i_1=0}^{d_{A_1}^2-1}\sum_{i_2=0}^{d_{A_2}^2-1}\sum_{i_3=0}^{d_{A_3}^2-1} C_{i_1i_2i_3} X_{i_1}^{(A_1)} \otimes X_{i_2}^{(A_2)} \otimes X_{i_3}^{(A_3)}, \quad (29)$$

with

$$C_{i_1i_2i_3} = Tr\left[X_{i_1}^{(A_1)} \otimes X_{i_2}^{(A_2)} \otimes X_{i_3}^{(A_3)}\right], \quad (30)$$

where

$$X_0^{(A_s)} = \frac{1}{\sqrt{d_{A_s}}}\mathbb{I}^{A_s} \quad , \quad X_{i\neq 0}^{(A_s)} = \frac{1}{\sqrt{2}}\lambda_i^{(A_s)}. \quad (31)$$

The A_s-correlation matrices are defined as follows

$$\mathcal{T}^{(A_1)} = \left\{\{C_{i_100}\},\{C_{i_1i_20}\},\{C_{i_1i_21}\},\{C_{i_1i_22}\},\{C_{i_1i_23}\},\{C_{i_10i_3}\}\right\}, \quad (32)$$

$$\mathcal{T}^{(A_2)} = \left\{\{C_{0i_20}\},\{C_{i_1i_20}\}^t,\{C_{i_1i_21}\}^t,\{C_{i_1i_22}\}^t,\{C_{i_1i_23}\}^t,\{C_{0i_2i_3}\}\right\}, \quad (33)$$

$$\mathcal{T}^{(A_3)} = \left\{\{C_{i_100}\},\{C_{i_10i_3}\}^t,\{C_{i_11i_3}\}^t,\{C_{i_12i_3}\}^t,\{C_{i_13i_3}\}^t,\{C_{0i_2i_3}\}^t\right\}, \quad (34)$$

and the quantum discord corresponding to A_s is

$$Q_{A_s} = \left\|\mathcal{T}^{(A_s)} - \tilde{P}_{A_s}\mathcal{T}^{(A_s)}\right\|_2^2, \quad (35)$$

where $\tilde{P}_{A_1}$ is the projective operator which minimizes Eq. (35). Then $\tilde{P}_{A_s}$ should be applied to all A_s-correlation matrices to produce a new correlation matrix $C_{i_1i_2i_3}$. This procedure should be repeated for all subsystems to capture the total quantum correlation which is defined as follows

$$Q_{A_1A_2A_3} = \max_{\mathcal{P}}\left\{Q_{A_{i_1}A_{i_2}A_{i_3}}\right\}, \quad (36)$$

where $\mathcal{P}$ means that we should maximize over all 3! possible permutations of $\{A_1A_2A_3\}$. We will use the simple function $f = \frac{\sqrt{2}}{2}$ for this measure, as we did for $\bar{Q}_{A_i|A_jA_k}$.

4 Quantum correlations of the three-flavored neutrino oscillating system

In order to study the quantum correlation measures, we should find the density matrices corresponding to different initial neutrinos which are defined as follows

$$\rho_\alpha(t) = |\nu_\alpha(t)\rangle\langle\nu_\alpha(t)|. \quad (37)$$

Using the eigenstate representations $|1\rangle = \binom{1}{0}$ and $|1\rangle = \binom{0}{1}$ we have

$$\rho_\alpha(t) = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & A_{\alpha e}A_{\alpha e}^* & 0 & A_{\alpha e}A_{\alpha\mu}^* & A_{\alpha e}A_{\alpha\tau}^* & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & A_{\alpha\mu}A_{\alpha e}^* & 0 & A_{\alpha\mu}A_{\alpha\mu}^* & A_{\alpha\mu}A_{\alpha\tau}^* & 0 \\ 0 & 0 & 0 & A_{\alpha\tau}A_{\alpha e}^* & 0 & A_{\alpha\tau}A_{\alpha\mu}^* & A_{\alpha\tau}A_{\alpha\tau}^* & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}. \quad (38)$$

When electron neutrino is generated initially, the oscillation probabilities, i.e. $P_{ee} = A_{ee}^2$, $P_{e\mu} = A_{e\mu}^2$ and $P_{e\tau} = A_{e\tau}^2$, can be easily found by Eq. (11). Here we will use the latest results obtained for the oscillation parameters of neutrinos in the normal ordering of the neutrino mass spectrum ($m_1 < m_2 < m_3$) as follows [24]

$$\theta_{12} = 33.41°, \tag{39}$$

$$\theta_{13} = 8.54°, \tag{40}$$

$$\theta_{23} = 49.1°, \tag{41}$$

$$\delta = 197°, \tag{42}$$

$$\Delta m^2_{21} = -\Delta m^2_{12} = 7.41 \times 10^{-5}, \tag{43}$$

$$\Delta m^2_{31} = -\Delta m^2_{13} = 2.511 \times 10^{-3}, \tag{44}$$

$$\Delta m^2_{32} = -\Delta m^2_{23} = 2.511 \times 10^{-3}. \tag{45}$$

The oscillation probabilities for electron neutrino as the initial particle are plotted in Fig. 1 as functions of $L/2E$. In this case, the survival probability is $P_{ee} = |A_{ee}|^2$ and never vanishes while the transition probabilities, $P_{e\mu} = |A_{e\mu}|^2$ and $P_{e\tau} = |A_{ee}|^2$ are zero in the beginning and vanish periodically.

Using Eq. (20), it can be shown that the concurrence fill has the following relationship with the oscillation probabilities [16]

$$F_{A_1A_2A_3} = 8\sqrt[4]{\frac{P^2_{ee}P^2_{e\mu}P^2_{e\tau}\left[P_{e\tau}P_{e\mu} + P_{ee}\left(P_{e\mu} + P_{e\tau}\right)\right]}{3}}. \tag{45}$$

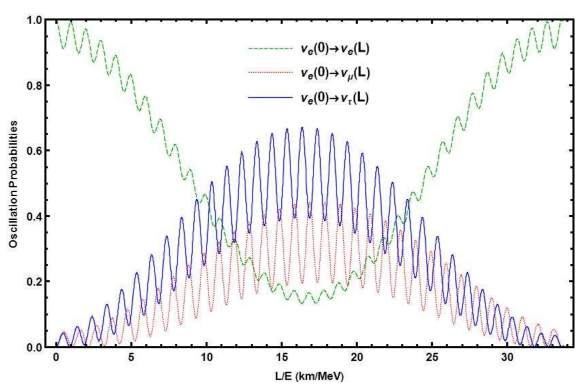

Figure 1. Oscillation probabilities in the single-particle approach of neutrino oscillations as functions of $L/2E$, including $P(\nu_e \to \nu_e)$ (green, dashed line) $P(\nu_e \to \nu_\mu)$ (red, dotted line) and $P(\nu_e \to \nu_\tau)$ (blue, solid line)

Having Eq. (38), it is easy to find the corresponding 3×3 correlation matrices $\mathcal{T}^{(A_1)}$, $\mathcal{T}^{(A_2)}$, and $\mathcal{T}^{(A_3)}$ from Eqs. (32)-(34), respectively. Finding the $\mathcal{T}^{A_s}(\mathcal{T}^{A_s})^\dagger$ matrices, it is straightforward to find their eigenvalues which give the following relation for $\overline{\mathcal{Q}}_{A_1A_2A_3}$

$$\overline{\mathcal{Q}}_{A_i|A_jA_k} = \frac{4}{3}\left[P_{ee}\left(P_{e\mu} + P_{e\tau}\right) + P_{e\mu}(P_{ee} + P_{e\tau}) + P_{e\tau}\left(P_{ee} + P_{e\mu}\right)\right]. \tag{46}$$

To find the total quantum correlation measure, we should start with one subsystem A_s, calculate $\mathcal{Q}_{A_s}$ and $\widetilde{P}_{A_s}$, then do the corresponding projection on the correlation matrix. This procedure will be repeated for the remaining subsystems, one after another. The maximization in Eq. (36) should be done on all 3! different permutations of $\{A_1A_2A_3\}$, which leads to a piecewise function. However, for simplicity, we express it as follows

$$\mathcal{Q}_{A_1A_2A_3} = 4\left(P_{ee}P_{e\mu} + P_{ee}P_{e\tau} + P_{e\mu}P\_e\tau\right). \tag{47}$$

The average quantum correlation ($\overline{\mathcal{Q}}_{A_i|A_jA_k}$), total quantum correlation ($\mathcal{Q}_{A_1A_2A_3}$) and concurrence fill ($F_{A_1A_2A_3}$) are shown in Fig. 2 as functions of $L/2E$. At the production point ($L/2E = 0$) the measures have zero values, as expected. Since all of the measures depend on oscillation probabilities, they show periodic behavior. However, none of the measures suffer nonanalytic sharp peaks. Quantum discord measures contain more information, especially $\mathcal{Q}_{A_1A_2A_3}$ which reaches the maximum possible value at some points.

Examining the problem in the wave packet approach provides an enlightening description of quantum decoherence induced during propagation by localization effects. We use the oscillation parameters in Eqs. (41)-(46) and $\frac{\sigma_p}{E} = 10^{-2}$ in our calculations. The survival and transition probabilities are plotted logarithmically in Fig. 3 as functions of $L/2E$. After an initial almost stable trend, both survival and transition probabilities start a fluctuating intermediate phase which ends in a final stationary value. Except for a small interval, the survival probability is greater than the transition probabilities. In addition, most of the time the probability of electron neutrinos transition into tau neutrinos is higher than the probability of the transition into muon neutrinos.

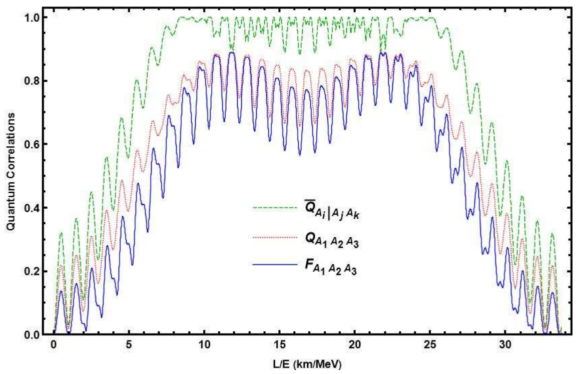

Figure 2. Quantum correlations in the single-particle approach of neutrino oscillations as functions of $\boldsymbol{L/2E}$, including $\boldsymbol{\overline{Q}_{A_i|A_jA_k}}$ (green, dashed line) $\boldsymbol{Q_{A_1A_2A_3}}$ (red, dotted line) and $\boldsymbol{F_{A_1A_2A_3}}$ (blue, solid line)

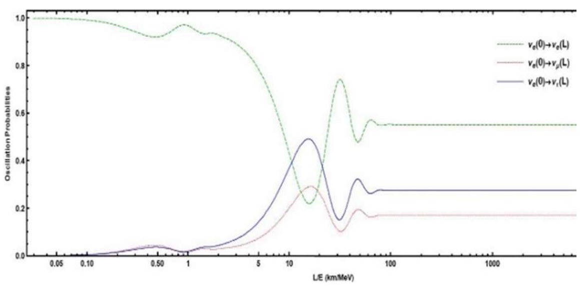

Figure 3. Oscillation probabilities in the wave packet approach of neutrino oscillations as functions of $L/2E$, including $P(\nu_e \to \nu_e)$ (green, dashed line) $P(\nu_e \to \nu_\mu)$ (red, dotted line) and $P(\nu_e \to \nu_\tau)$ (blue, solid line).

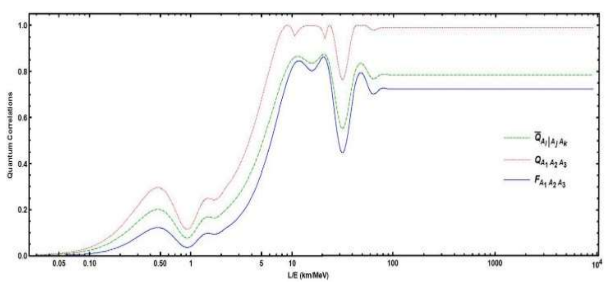

Figure 4. Quantum correlations in the wave packet approach of neutrino oscillations as functions of $L/2E$ including $\overline{Q}_{A_i|A_jA_k}$ (green, dashed line) $Q_{A_1A_2A_3}$ (red, dotted line) and $F_{A_1A_2A_3}$ (blue, solid line).

Figure 4, contains the logarithmic plots of average quantum correlation ($\overline{Q}_{A_i|A_jA_k}$), total quantum correlation ($Q_{A_1A_2A_3}$), and concurrence fill ($F_{A_1A_2A_3}$), as functions of $L/2E$ for the wave packet approach. Again we can see two different phases in the diagrams, including the initial fluctuating phase and the final steady one. Just like the single-particle approach, quantum discord measures contain more information than the entanglement, especially $Q_{A_1A_2A_3}$ which has larger values than other measures in the whole graph.

So far, we have studied the quantum correlations created due to neutrino oscillations in the general case of three flavors for an electron neutrino as the initial particle. Examining the states that occur with initial muon or tau neutrinos similar results are provided, except for a few computational details. Therefore, to avoid prolonging the discussion, let us refrain from examining these cases.

5 Conclusions

In this paper we have studied the quantum correlations of a system of three-flavor oscillating neutrinos, using a measure of entanglement which was recently introduced as the best entanglement measure to survey this system [16], and two measures of quantum discord. In order to consider the localization effects, both single-particle and wave packet approaches were considered. In the single-particle approach, the oscillation pattern is repeated, while in the wave packet scenario, this oscillation pattern disappears after a while, but the quantum correlation remains nonzero. It is interesting to find that the quantum properties of the system do not vanish even in large-scale distances that one would expect to observe the classical properties due to the decoherence effects. In addition, the compatibility of entanglement and discord-like measure is revisited for this system.

Moreover, it was shown that the quantum discord measures contain more quantum resources than the entanglement measures. Likewise, the measure of total quantum correlation ($Q_{A_1A_2A_3}$) has always greater values, while behaves smoothy and also stable which makes it superior to the other measures.

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